School of Isolated and Distance Education
MATHEMATICS SPECIALIST Year 11

Test 3 2023

Section 1: Calculator Free

Time allowed for this section

Working time: 20 minutes

Mark allocation: 29 marks

PERMISSIBLE ITEMS

Standard Items: pens, pencils, pencil sharpener, highlighter, eraser, ruler
Special Items: none

STANDARD FORMULAE SHEET IS PROVIDED
NO OTHER ITEMS MAY BE TAKEN INTO THE EXAMINATION ROOM

INSTRUCTIONS FOR CANDIDATES

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

All work must be done in the space provided. Should you need extra working area you may use the blank pages at the end.

QUESTION 1   [1, 2, 3, 4 = 10 marks]

(a)   Evaluate $\dfrac{20!}{18!\,2!}$

(b)   Simplify $\dfrac{n!}{(n-2)!}$

(c)   Find the value of n, if $^nC_2 = 3$

(d)   Solve for n, if $^nP_4 : {}^nC_2 = 12 : 1$

QUESTION 2   [2, 3 = 5 marks]

The digital sum of a natural number is defined to be the sum of its digits. For example, the digital sum of 123 is $1 + 2 + 3 = 6$

(a) Nineteen two-digit numbers are selected. Prove that at least two of them have the same digital sum

(b) Supposed that 82 three digital numbers are selected. Prove that at least four of them have the same digital sum.

QUESTION 3   [1, 3 = 4 marks]

By definition, the kth term in row n of Pascal's Triangle, given as $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$.

(a) With $n = 5$ and $k = 2$, show that the left and right sides of the identity are equal.

(b) Prove the identity is always true, subject to restrictions $n, k \in Integers, n \geq k \geq 1$.

QUESTION 4   [1, 1, 1, 3 = 6 marks]

Use Pascal's triangle provided at the end of this test to answer the following.

How many groups of six can be chosen from five women and four men at a workplace given:

(a)   There is no restriction on who is in the group

(b)   The workplace needs four women and two man in the group

(c)   The group contains at most one man?

(d)   The group contains at least two woman?

QUESTION 5   [2, 2 = 4 marks)

A line of Pascal's triangle is shown below.

1	9	36	84	126	126	84	36	9	1

With reference to that line, demonstrate the truth (or otherwise) of these general statements.

(a)   (i) $^{n}C_{r} = {}^{n}C_{n-r}$

        (ii) $^{n}C_{r} = 2^{n}C_{n-r}$

End of Calculator Free section.

Pascal's Triangle

Left triangle:
```
                1
              1   1
            1   2   1
          1   3   3   1
        1   4   6   4   1
      1   5   10  10  5   1
```

Right triangle (combinatorial notation):

$$^0C_0$$
$$^1C_0 \quad ^1C_1$$
$$^2C_0 \quad ^2C_1 \quad ^2C_2$$
$$^3C_0 \quad ^3C_1 \quad ^3C_2 \quad ^3C_3$$
$$^4C_0 \quad ^4C_1 \quad ^4C_2 \quad ^4C_3 \quad ^4C_4$$
$$^5C_0 \quad ^5C_1 \quad ^5C_2 \quad ^5C_3 \quad ^5C_4 \quad ^5C_5$$

							1									
						1		1								
					1		2		1							
				1		3		3		1						
			1		4		6		4		1					
		1		5		10		10		5		1				
	1		6		15		20		15		6		1			
1		7		21		35		35		21		7		1		
1	8		28		56		70		56		28		8		1	
1	9	36		84		126		126		84		36		9		1
1	10	45	120		210		252		210		120		45	10		1
1	11	55	165	330		462		462		330		165	55	11		1
1	12	66	220	495	792		924		792		495	220	66	12	1	
1	13	78	286	715	1287	1716		1716		1287	715	286	78	13	1	
1	14	91	364	1001	2002	3003	3432	3003	2002	1001	364	91	14	1		
1	15	105	455	1365	3003	5005	6435	6435	5005	3003	1365	455	105	15	1	
1	16	120	560	1820	4368	8008	11440	12870	11440	8008	4368	1820	560	120	16	1

Additional page for working out

School of Isolated and Distance Education
MATHEMATICS SPECIALIST Year 11

Test 3 2023

Section 2: Calculator assumed

Time allowed for this section

Working time: 35 minutes

Mark allocation: 38 marks

PERMISSIBLE ITEMS

Standard Items: pens, pencils, pencil sharpener, highlighter, eraser, ruler
Special Items: **Formulae Sheet, CAS calculator, ONE A4 page of notes**

STANDARD FORMULAE SHEET IS PROVIDED
NO OTHER ITEMS MAY BE TAKEN INTO THE EXAMINATION ROOM

INSTRUCTIONS FOR CANDIDATES
Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

All work must be done in the space provided. Should you need extra working area you may use the blank pages at the end.

Student's name:_____

SIDE Teacher's name: _____

SUPERVISOR'S DECLARATION
I declare that this test paper has been completed without assistance by the student named above. The time and resource restrictions have been observed and the student has NOT accessed additional notes other than the one A4 page allowed, texts, reference books, the internet, a computer, a mobile phone or other electronic device. I understand that this paper will not be counted for assessment if these conditions have not been met and that notifications will occur.

Supervisor's name: _____

Supervisor's signature:_____ **Date:**_____

QUESTION 6   **[1, 2, 4 = 7 marks]**

(a) How many 5-character passwords can be created from the lower-case letters of the alphabet without repetition?

(b) How many 5-character passwords can be created from the lower-case letters of the alphabet, without repetition, that contain exactly three vowels?

(c) How many 8-character passwords can be created from the lower-case letters of the alphabet and the digits 0 to 9, without repetition, that contain exactly three vowels OR exactly three consonants?

QUESTION 7   **[1, 2, 2 = 5 marks]**

How many ways can two blue, three black and four green marbles to be arranged in a row:

   a) Without restriction

   b) If the first and the last are blue

   c) If three black are next to each other?

QUESTION 8 [6, 1, 2, 2 = 11 marks]

(a) A box contains 400 balls, each of which is blue, red, green, yellow or orange. The ratio of blue to red to green balls is 1 : 4 : 2. The ratio of green to yellow to orange balls is 1:3:6. What is the smallest number of balls that must be drawn to ensure that at least 50 balls of one colour are selected?

(b) Consider the letters in the word CULLACAABARDEE, an Aboriginal Noongar word meaning meeting place. Determine the number of different:

(i) Combinations of 4 letters chosen from the consonants in the word.

(ii) How many ways can we arrange this word?

(iii) How many subsets of at least 1 letter can be formed from the word KAYA?

QUESTION 9 [6, 1, 1 = 8 marks]

In a group of 1030 students:

- 810 studied exactly one of Mathematics, Japanese and Health.
- The number of students studying Health is 200 less than the number of students studying Mathematics and 250 less than the number of students studying Japanese.
- 60 students study Mathematics and Japanese and of these 10 do not study Health.
- One quarter of Health students also study Japanese and 20 of these do not study Mathematics.

(a) How many students study Mathematics only?

(b) How many students study none of these subjects?

(c) How many students study exactly two subjects?

QUESTION 10   [1, 1, 3, 1, 1 = 7 marks]

Consider the set of integers 1 and 96 inclusive. Let sets A and B consists of those integers that are multiples of 6 and 8 respectively.

(a) What is the lowest common multiple of 6 and 8?

(b) How many integers belong $A \cap B$?

(c) How many integers are divisible by 6 or 8?

(d) How many integers that are not divisible by 6 and 8?

(e) An integer is chosen at random, what is the probability that it is not divisible by 6 or 8?

End of Test

Additional page for working out.